

Inverse and Joint Variation

Does the data in the table suggest that y varies inversely as x ? If so, find the constant of variation and the equation.

$$1. \begin{array}{c|c|c|c|c|c} x & 1 & 2 & 3 & 6 & 12 \\ \hline y & 6 & 3 & 2 & 1 & \frac{1}{2} \end{array}$$

$$2. \begin{array}{c|c|c|c|c|c} x & 40 & 20 & 10 & 8 & 4 \\ \hline y & 1 & 2 & 4 & 5 & 10 \end{array}$$

State whether each formula expresses inverse variation or joint variation, and state the constant of variation.

$$3. E = \frac{360}{n}$$

$$4. S = \frac{1}{2}gt^2$$

$$5. 12 = bh$$

In Exercises 6–7, y varies inversely as x .

$$6. \text{ If } y = 6 \text{ when } x = 2, \text{ find } y \text{ when } x = 4. \quad \underline{\hspace{2cm}}$$

$$7. \text{ If } y = 5 \text{ when } x = 2, \text{ find } y \text{ when } x = 6. \quad \underline{\hspace{2cm}}$$

In Exercises 8–9, x varies inversely as y^2 .

$$8. \text{ If } x = 9 \text{ when } y = 2, \text{ find } x \text{ when } y = 3. \quad \underline{\hspace{2cm}}$$

$$9. \text{ If } x = 4 \text{ when } y = 5, \text{ find } y \text{ when } x = 2. \quad \underline{\hspace{2cm}}$$

In Exercises 10–11, z varies jointly as x and y .

$$10. \text{ If } z = 18 \text{ when } x = 2 \text{ and } y = 9, \text{ find } z \text{ when } x = 3 \text{ and } y = 9. \quad \underline{\hspace{2cm}}$$

$$11. \text{ If } z = 60 \text{ when } x = 6 \text{ and } y = 8, \text{ find } z \text{ when } x = 10 \text{ and } y = 12. \quad \underline{\hspace{2cm}}$$

In Exercises 12–13, use k as the constant of variation and write an equation for the variation.

$$12. \text{ The area } A \text{ of a triangle varies jointly as its height } h \text{ and its base } b. \quad \underline{\hspace{2cm}}$$

$$13. r \text{ varies jointly as } s \text{ and } t \text{ and inversely as the cube of } v. \quad \underline{\hspace{2cm}}$$

State whether each equation represents inverse variation, joint variation, or combined variation.

$$1. xy = 2$$

$$2. z = kxy$$

$$3. y = \frac{x}{z}$$

$$4. t = k \frac{r^2}{s}$$

$$5. V = \frac{1}{3}\pi r^2 h$$

$$6. F = kdg^2$$

$$7. \text{ If } z \text{ varies inversely as } t, \text{ and } t = 10 \text{ when } z = 4, \text{ find the constant of variation and write an equation for the variation.}$$

$$8. \text{ If } r \text{ varies inversely as } s, \text{ and } s = 4 \text{ when } r = 12, \text{ find } s \text{ when } r = 6.$$

$$9. \text{ If } x \text{ varies jointly as } y \text{ and the square of } z, \text{ and } x = 15 \text{ when } y = 5 \text{ and } z = 1, \text{ find } x \text{ when } y = 10 \text{ and } z = 2.$$

PRACTICE EXERCISES

Does the data in the table suggest that y varies inversely as x ? If so, find the constant of variation and the equation.

1. x	1	2	4	8	12
y	4	2	1	$\frac{1}{2}$	$\frac{1}{3}$

2. x	50	25	20	10
y	2	4	5	10

State whether the formula expresses inverse variation, joint variation, or neither. State the constant of variation.

3. $V = e^3$

4. $24 = lw$

5. $xy = 18$

6. $p = 3s$

7. $V = \pi r^2 h$

8. $A = \frac{1}{2}bh$

In Exercises 9–12, y varies inversely as x .

9. If $y = 4$ when $x = 2$, find y when $x = 6$.
10. If $y = 8$ when $x = 3$, find y when $x = 12$.
11. If $y = 20.4$ when $x = -6.8$, find x when $y = 47.6$.
12. If $y = 82.0$ when $x = 32.8$, find x when $y = 180.4$.

In Exercises 13–16, x varies inversely as y^2 .

13. If $x = 4$ when $y = 2$, find x when $y = 3$.
14. If $x = 6$ when $y = 4$, find x when $y = 8$.
15. If $x = 42$ when $y = 21$, find y when $x = 378$.
16. If $x = 72$ when $y = 27$, find y when $x = 54$.

In Exercises 17–22, z varies jointly as x and y .

17. If $z = 12$ when $x = 3$ and $y = 4$, find z when $x = 5$ and $y = 6$.
18. If $z = 15$ when $x = 5$ and $y = 2$, find z when $x = 7$ and $y = 4$.
19. If $z = 198$ when $x = 33$ and $y = 9$, find z when $x = 24$ and $y = 36$.
20. If $z = 1088$ when $x = 34$ and $y = 4$, find z when $x = 28$ and $y = 18$.
21. If $z = 1.1$ when $x = 55$ and $y = 2$, find z when $x = 75$ and $y = 3$.
22. If $z = 742.5$ when $x = 33$ and $y = 15$, find z when $x = 12$ and $y = 15$.

In Exercises 23–30, use k as the constant of variation and write an equation for the variation.

23. t varies directly as q and inversely as s .
24. w varies jointly as x and y .
25. r varies directly as the square of t and inversely as the cube of v .
26. g varies jointly as a and b and inversely as the square of c .
27. The weight of an object varies inversely as the square of the distance of the object from the center of the earth.
28. The load that a beam of constant length can support varies jointly as its width and the square of its height.
29. The resistance of a wire to the passage of an electric current varies directly as the length of the wire and inversely as the square of its diameter.
30. The volume of a right circular cylinder varies jointly as its height and the

31. If y varies inversely as x , and $y = 42$ when $x = 3\frac{1}{2}$, find y when $x = 1\frac{2}{5}$.
32. If y varies inversely as the square of x , and $y = 50$ when $x = 4$, find y when $x = 5$.
33. If w varies jointly as x and y , and $w = 28$ when $x = 4$ and $y = 21$, find w when $x = 12$ and $y = 17$.
34. If c varies jointly as d and the square of e , and $c = 30$ when $d = 15$ and $e = 2$, find d when $c = 6$ and $e = 8$.
35. If y varies directly as x and inversely as z , and $y = 49$ when $x = 14$ and $z = 4$, find y when $x = 16$ and $z = 7$.
36. If a varies directly as b and inversely as the square of c , and $a = 46$ when $b = 12$ and $c = 6$, find b when $a = 23$ and $c = 6$.
37. If x varies directly as the square of y , and $x = 4$ when $y = \frac{1}{2}$, find x when $y = \frac{1}{4}$.
38. If a varies jointly as b and c , and $a = 100$ when $b = 10$ and $c = 5$, find c when $a = 150$ and $b = 15$.
39. If d varies jointly as r and t , and $d = 110$ when $r = 55$ and $t = 2$, find r when $d = 40$ and $t = 3$.
40. If y varies directly as x and inversely as z^2 , and $x = 48$ when $y = 8$ and $z = 3$, find x when $y = 12$ and $z = 2$.

In Exercises 1–3, y varies inversely as x .

1. If $y = 3$ and $x = 4$, find y when $x = 2$. _____
2. If $y = 1$ and $x = 3$, find x when $y = 6$. _____
3. If $x = 10$ when $y = \frac{1}{2}$, find y when $x = 5$. _____

In Exercises 4–5, y varies jointly as x and z .

4. If $z = 3$ when $x = 3$ and $y = 6$, find y when $x = 2$ and $z = 3$. _____
5. If $y = 2$ when $x = 6$ and $z = 5$, find y when $x = 1$ and $z = 10$. _____